Chapter 6: Challenge Solutions

Challenge:

The analog transfer function of a 2[nd] order LPF is:

$$H(s) = \frac{1}{s^2 + (1/Q)s + 1}$$

The analog LPF has the following specifications: Q = 1, f_c = 1kHz, f_s = 44.1kHz

Apply the bi-linear transform and some algebra to find the coefficients. (Answer: a_0 = 0.0047 a_1 = 0.0095 a_2 = 0.0047 b_1 = -1.8485 b_2 0.8673)

Step 1: find ω_n

$$\omega_n = 2\pi f_c / f_s = 0.0714$$

Step 2: For the LPF, let s = s/ω_n

$$H(s) = \frac{1}{s^2 + (1/Q)s + 1}$$

$$= \frac{1}{\frac{s^2}{\omega_n^2} + \frac{s}{\omega_n} + 1}$$

$$= \frac{1}{\frac{s^2}{\omega_n^2} + \frac{s}{\omega_n} + 1} \frac{\omega_n^2}{\omega_n^2}$$

$$= \frac{\omega_n^2}{s^2 + s\omega_n + \omega_n^2}$$

$$= \frac{0.0051}{s^2 + 0.0714s + 0.0051}$$

$$\text{let s} = \frac{z-1}{z+1}$$

$$H(s) = \frac{0.0051}{\frac{(z-1)^2}{(z+1)^2} + 0.0714\frac{z-1}{z+1} + 0.0051}$$

$$= \frac{0.0051}{\frac{(z-1)^2}{(z+1)^2} + 0.0714\frac{z-1}{z+1} + 0.0051} \frac{(z+1)^2}{(z+1)^2}$$

$$= \frac{0.0051(z+1)^2}{(z-1)^2 + 0.0714(z-1)(z+1) + 0.0051(z+1)^2}$$

$$= \frac{0.0051z^2 + 0.0102z + 0.0051}{z^2 - 2z + 1 + 0.0714z^2 - 0.0714 + 0.0051z^2 + 0.0102z + 0.0051}$$

$$= \frac{0.0051z^2 + 0.0102z + 0.0051}{1.0765z^2 - 1.9898z + 0.9337}$$

$$= \frac{0.0051z^2 + 0.0102z + 0.0051}{1.0765z^2 - 1.9898z + 0.9337} \frac{\frac{1}{z^2}}{\frac{1}{z^2}}$$

$$= \frac{0.0051 + 0.0102z^{-1} + 0.0051z^{-2}}{1.0765 - 1.9898z^{-1} + 0.9337z^{-2}}$$

normalize to force the first term of the denominator to be 1.0

$$= \frac{0.0051 + 0.0102z^{-1} + 0.0051z^{-2}}{1.0765 - 1.9898z^{-1} + 0.9337z^{-2}} \frac{\frac{1}{1.0765}}{\frac{1}{1.0765}}$$

$$= \frac{0.0047 + 0.0095z^{-1} + 0.0047z^{-2}}{1.0 - 1.8484z^{-1} + 0.8673z^{-2}}$$

$a_0 = 0.0047$
$a_1 = 0.0095$
$a_2 = 0.0047$
$b_1 = -1.8484$
$b_2 = 0.8673$